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Heave and Pitch Dynamics of Shallow Draft Surface Vehicles

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14. ABSTRACT

This project's objective is to develop a model of the heave and pitch dynamics for a shallow draft surface vehicle. The model developed uses a novel approach of representing the ship dynamics in the frequency domain. The model is based on theoretical first-principles involving highly nonlinear equations that depend on the water wave frequency. Compared to time domain modeling and simulation approaches, the frequency domain model provides greater insight to the surface vehicle's behavior over a wider operating range. To validate the model, experimental data from towing tank experiments with a 3 meter, shallow draft boat is compared to theoretical model results. The experimental data compares well with the model.

15. SUBJECT TERMS

Heave	Dynamics	Surface vessel
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Nomenclature

ζ	Free surface displacement of wave
ζ_a	Wave amplitude
ω_w	Wave frequency
ω_e	Frequency of encountered wave
K	Wave number
V	Vehicle velocity
μ	Relative heading of vehicle to wave train
z	Heave displacement
z_a	Heave amplitude
ϕ_z	Heave phase angle relative to wave
z_{ss}	Heave steady state displacement
θ	Pitch displacement
θ_a	Pitch amplitude
θ_{ss}	Pitch steady state displacement
ϕ_θ	Pitch phase angle relative to wave
m	Vehicle mass
a_z	Added mass (the mass of water excited by the vehicle's motion)
b	Heave damping coefficient
c	Heave stiffness coefficient
d	Pitch inertia coupling term
e	Pitch damping coupling term
h	Pitch stiffness coupling term
I_{yy}	Pitch axis moment of inertia
A_{yy}	Added mass of inertia
B	Pitch damping coefficient
C	Pitch stiffness coefficient
D	Heave inertia coupling term
E	Heave damping coupling term
H	Heave stiffness coupling term
g	Acceleration of gravity
ρ	Density of water
k_a	Added mass coefficient
R_z	Ratio of radiated wave amplitude to heave motion amplitude
B_n	Vehicle's sectional beam width

z_{ref}	Sectional reference draft of the wetted surface
a_n	Sectional added mass
b_n	Sectional damping coefficient
c_n	Sectional stiffness coefficient
F	Wave induced force
F_0	Amplitude of force
ϕ_F	Phase angle of force relative to wave
F_s	Amplitude of in-phase force component
F_c	Amplitude of out-of-phase force component
f_s, f_c	Normalized amplitudes of force components
M	Wave induced moment
M_0	Amplitude of moment
ϕ_M	Phase angle of moment relative to wave
M_s	Amplitude of in-phase moment component
M_c	Amplitude of out-of-phase moment component
m_s, m_c	Normalized amplitudes of moment components
$\mathbf{M}$	Inertia matrix
$\mathbf{C}$	Damping matrix
$\mathbf{K}$	Stiffness matrix
$\vec{\mathbf{x}}$	State vector
$\vec{\mathbf{y}}$	Output vector
$\vec{\mathbf{u}}$	Periodic excitation vector
$\mathbf{\Gamma_A}$	System state matrix
$\mathbf{\Gamma_B}$	System input matrix
$\mathbf{\Gamma_C}$	System output matrix
$\mathbf{I}$	Identity matrix
s	Laplace transform variable
$\mathbf{G}(s)$	Transfer function matrix of heave and pitch dynamics
$G_{ZF}(s)$	Transfer function from force to heave
$G_{ZM}(s)$	Transfer function from moment to heave
$G_{\theta F}(s)$	Transfer function from force to pitch
$G_{\theta M}(s)$	Transfer function from moment to pitch
$G_{F\zeta}(s)$	Transfer function from wave to force

$G_{M\zeta}(s)$	Transfer function from wave to moment
$G_{Z\zeta}(s)$	Transfer function from wave to heave
$G_{\theta\zeta}(s)$	Transfer function from wave to pitch
$\lvert G_{Z\zeta}(j\omega_e)\rvert$	Magnitude of transfer function $G_{Z\zeta}(s)$ at frequency ω_e
$\angle G_{Z\zeta}(j\omega_e)$	Phase of transfer function $G_{Z\zeta}(s)$ at frequency ω_e
$\lvert G_{\theta\zeta}(j\omega_e)\rvert$	Magnitude of transfer function $G_{\theta\zeta}(s)$ at frequency ω_e
$\angle G_{\theta\zeta}(j\omega_e)$	Phase of transfer function $G_{\theta\zeta}(s)$ at frequency ω_e

1.0 Introduction

The motivation for this research is to enhance a first principles, physics based, theoretical model to replicate small planing boat motions as described in Cortesi, et. al. (2008) and Cortesi and Justh (2007). This model is based on semi-empirical studies of planning hulls, such as those developed by Savitsky and Gore (1979) as well as the principles discussed in Fossen (1994) regarding the modeling of displacement hulls. There has been some success in using this relatively simple model; but it was deemed desirable to enhance the features of the model by adding the effects of periodic wave action on the small boat. Some initial success was achieved in this endeavor by using strip theory, as described in McCormick (1973) and Bhattacharyya (1978). However, the first-principles model only worked for a limited range of boat speeds and wave frequencies. It was soon realized that for all of its simplicity, the first model was unable to incorporate the complex effects of wave action. As a result, the NRL model was supplemented for one which allows incorporation of the non-linear effects of a small boat undergoing motions due to wave action as discussed in McCormick (1973), Bhattacharyya (1978), and McCormick (2010). Additionally, an approach was needed which allows analytical data to be compared to experimental data over the full range of encountering frequencies, ω_e . It was therefore decided to use a frequency based approach. Using this approach, the validity of the analytical model was able to be determined quickly and effectively.

This paper is organized into six sections. In Section 1.0, a brief introduction is given. Section 2.0 presents the fundamentals of wave induced planar motions. Section 3.0 discusses the coupled equations of motion for heave and pitch, and discusses the methods used to determine the coefficients for the equations of motion and the derivation of the necessary transfer functions. It goes on to discuss the derivation of the vehicle parameters and the derivation of the wave induced force and moment parameters. Section 4.0 discusses the methodology used to conduct the validation experiment. Section 5.0 contains the results of the model and the experiment. In Section 6.0, a summary is given as well as recommendations for future work.

2.0 Review of Wave-Induced Planar Motion Analysis

For this work, it is assumed that the vehicle is heading into the waves in a direction that is transverse to the wave crest line. The seaway consists of regular harmonic waves, and the lateral vehicle motions are neglected. Heave and pitch motions are referenced to the vehicle's body axis that is rigidly located at the vehicle's center of gravity, CG. The planar geometry of the vehicle is illustrated in Figure (1). The heaving displacement, z, is the vertical displacement of the CG from the still waterline; while pitch, θ, is the angular displacement of the vehicle's x-y plane from the still waterline. The free surface amplitude, ζ, is the vertical displacement of the water surface from the still waterline measured at the vehicle's CG.

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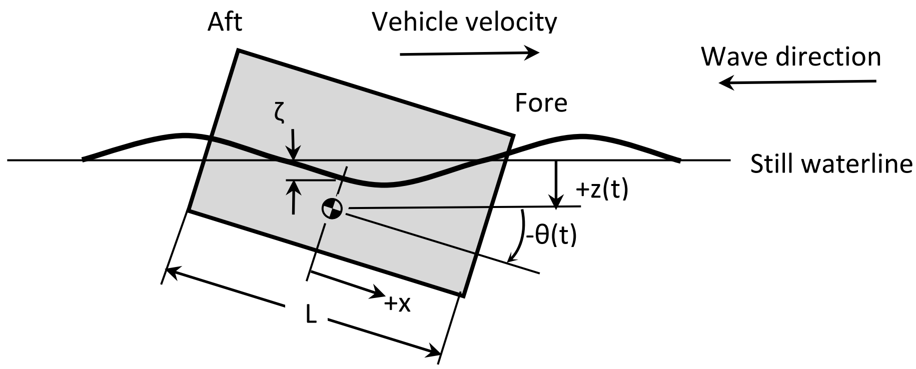

Fig. 1 Heaving and Pitching Motion in Waves

The effective free-surface displacement encountered by the vehicle as it travels is described by

$$\zeta = \zeta_a \sin(\omega_e t) \tag{1}$$

where ζ_a is the wave amplitude and ω_e is the frequency of encounter. This frequency is related to the vehicle's velocity, V, the relative heading to the wave, μ, and the wave frequency, ω_w,according to the relationship

$$\omega_e = \omega_w \left(1 - \frac{\omega_w V}{g} \cos \mu\right) \tag{2}$$

For the case where the vehicle is heading into the wave train, the encountering angle μ is 180^o.

The vehicle's wave-induced heave and pitch motions are governed by the vehicle's inertial interaction with the hydrostatic and hydrodynamic forces acting on it. These interactions can be represented by two, coupled, second-order differential equations with periodic forcing functions, as discussed in McCormick (1973), Bhattacharyya (1978) and McCormick (2010),

$$(m + a_z)\ddot{z} + b\dot{z} + cz + d\ddot{\theta} + e\dot{\theta} + h\theta = F(t) \tag{3.a}$$

$$(I_{yy} + A_{yy})\ddot{\theta} + B\dot{\theta} + C\theta + D\ddot{z} + E\dot{z} + Hz = M(t) \tag{3.b}$$

The coefficients in Eqs. (3.a) and (3.b) correspond to the inertia, damping, and equivalent stiffness properties associated with the vehicle-wave interaction. These coefficients are a function of the vehicle's particular hull shape, and are defined in Section 5.0. The terms $F(t)$ and $M(t)$ are the respective wave-induced excitation force and moment. These terms will be discussed Section 6.0. Both the system coefficients and the excitation terms are dependent on the encountered wave frequency, ω_e. In addition, the excitation terms also depend on the encountered wave amplitude, ζ_a. Thus, the heave and pitch dynamics are nonlinear in terms of frequency.

However, if we constrain our discussion to a particular encountering frequency, ω_e, of interest the system coefficients are constant and Eqs. (3.a) and (3.b) can be considered linear with regards to that frequency. Therefore, Eq. (3) can be conveniently cast in compact matrix form, as in Goldstein (2001), Meirovitch (2010) and Benaroya (2004).

$$\mathbf{M}(\omega_e)\begin{bmatrix}\ddot{z}\\ \ddot{\theta}\end{bmatrix}+\mathbf{C}(\omega_e)\begin{bmatrix}\dot{z}\\ \dot{\theta}\end{bmatrix}+\mathbf{K}(\omega_e)\begin{bmatrix}z\\ \theta\end{bmatrix}=\vec{\mathbf{u}}(\omega_e,t) \tag{4}$$

where the matrices are defined as follows:

The inertia matrix is

$$\mathbf{M}(\omega_e)=\begin{bmatrix}(m+a_z) & d\\ D & (I_{yy}+A_{yy})\end{bmatrix}$$

while the damping matrix is

$$\mathbf{C}(\omega_e)=\begin{bmatrix}b & e\\ E & B\end{bmatrix}$$

and the stiffness matrix is

$$\mathbf{K}(\omega_e)=\begin{bmatrix}c & h\\ H & C\end{bmatrix}.$$

Also, in Eq. (4) is the periodic excitation vector,

$$\vec{\mathbf{u}}(\omega_e,t)=\begin{bmatrix}F(\omega_e,t)\\ M(\omega_e,t)\end{bmatrix}.$$

To analyze the relationship between the wave induced forces and the heaving and pitching motion, the two second-order differential equations in Eq. (4) are formularized into a fourth-order, state variable model:

$$\dot{\vec{\mathbf{x}}}=\mathbf{\Gamma_A}\vec{\mathbf{x}}+\mathbf{\Gamma_B}\vec{\mathbf{u}} \tag{5.a}$$

and

$$\vec{\mathbf{y}}=\mathbf{\Gamma_C}\vec{\mathbf{x}} \tag{5.b}$$

where $\vec{\mathbf{x}}=\begin{bmatrix}z & \theta & \dot{z} & \dot{\theta}\end{bmatrix}^T$ is the state vector representing the entire state of the system at any given time, and $\vec{\mathbf{y}}=\begin{bmatrix}z & \theta\end{bmatrix}^T$ is output vector denoting the variables of interest, i.e., heave and pitch. The frequency dependent systems parameters are contained in the system matrices defined below.

$$\boldsymbol{\Gamma}_{\mathbf{A}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix},\quad \boldsymbol{\Gamma}_{\mathbf{B}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}, \text{ and } \boldsymbol{\Gamma}_{\mathbf{C}} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \tag{5.c}$$

3.0 Systems Analysis: Frequency Domain Model

Knowing that the heaving and pitching dynamics are driven by the periodic wave function in Eq. (2), it is highly advantageous to look at the wave-induced heave and pitch motions in the frequency domain. For a linear system, such as the one represented by Eq. (5), a sinusoidal input of a specific frequency results in an output that is also a sinusoid with the same frequency, but with a different amplitude and phase. The frequency-response function describes the amplitude change and phase shift as a function of frequency. Thus, for a given seaway characterized by Eq. (2) with a wave amplitude, ζ_a , and a frequency of encounter, ω_e, the resulting heaving and pitching motions will also be periodic with ω_e and characterized by amplitudes z_a, θ_a and phase angles ϕ_z, ϕ_θ as shown in Figure (2).

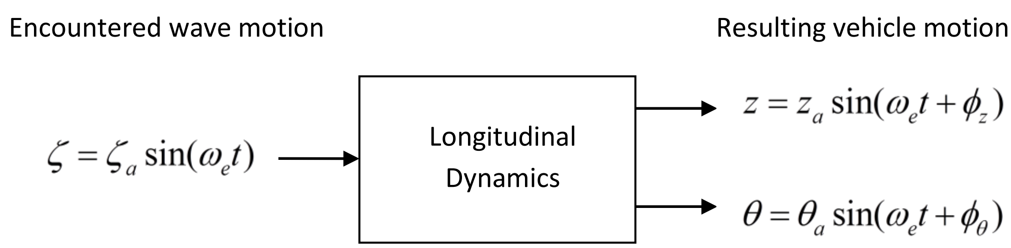

Fig. 2 Input/Output Relationship between Wave and Vehicle Motion

To obtain a frequency-response function, employ the transfer function concept which relates the system output to the system input in the Laplace domain. The transfer function concept allows one to convolve input/output relationships algebraically, as in Meirovitch (1985). In the Laplace domain, Eq. (5) can be written in a compact input-output relation as

$$\vec{\mathbf{y}}(s) = \mathbf{G}(s)\vec{\mathbf{u}}(s) \tag{6}$$

where $\vec{\mathbf{y}}(s)$ and $\vec{\mathbf{u}}(s)$ are the Laplace transforms of the output and input $\vec{\mathbf{y}}(t)$ and $\vec{\mathbf{u}}(t)$, respectively. The term $\mathbf{G}(s)$ is a transfer function matrix, which is an algebraic expression defined by the ratio

$$\mathbf{G}(s) = \frac{\vec{\mathbf{y}}(s)}{\vec{\mathbf{u}}(s)} \tag{7}$$

The transfer function matrix $\mathbf{G}(s)$ can be obtained from Eq. (5) by

$$\begin{aligned} \mathbf{G}(s) &= \boldsymbol{\Gamma}_{\mathbf{C}}\left[s\mathbf{I} - \boldsymbol{\Gamma}_{\mathbf{A}}\right]^{-1}\boldsymbol{\Gamma}_{\mathbf{B}} \\ &= \begin{bmatrix} G_{ZF}(s) & G_{ZM}(s) \\ G_{\theta F}(s) & G_{\theta M}(s) \end{bmatrix} \end{aligned} \tag{8}$$

The elements of the matrix $\mathbf{G}(s)$ are scalar transfer functions representing the interactions between heave (*z*), pitch (θ), the wave induced force (*F*) and moment (*M*) as defined by $G_{ZF}(s)=\dfrac{z(s)}{F(s)}$, $G_{ZM}(s)=\dfrac{z(s)}{M(s)}$, $G_{\theta F}(s)=\dfrac{\theta(s)}{F(s)}$, $G_{\theta M}(s)=\dfrac{\theta(s)}{M(s)}$, respectively. Therefore, the heave and pitch responses represented by Eq. (6) can be written more explicitly as

$$z(s)=G_{ZF}(s)F(s)+G_{ZM}(s)M(s) \tag{9}$$

$$\theta(s)=G_{\theta F}(s)F(s)+G_{\theta M}(s)M(s) \tag{10}$$

The relationship between the respective wave-induced force and moment and the encountered waves can also be expressed in the Laplace domain as

$$F(s)=G_{F\zeta}(s)\zeta(s) \tag{11}$$

$$M(s)=G_{M\zeta}(s)\zeta(s) \tag{12}$$

where transfer functions $G_{F\zeta}(s)=\dfrac{F(s)}{\zeta(s)}$ and $G_{M\zeta}(s)=\dfrac{M(s)}{\zeta(s)}$ represent the dynamics of the wave force and moment induced at the encountered waves. The derivation of the force and moment transfer functions, $G_{F\zeta}(s)$ and $G_{M\zeta}(s)$, is discussed in Section 6.0.

By substituting Eqs. (11) and (12) into Eqs. (9) and (10), the heaving and pitching motions are now related to the encountered waves by

$$z(s)=\left[G_{ZF}(s)G_{F\zeta}(s)+G_{ZM}(s)G_{M\zeta}(s)\right]\zeta(s) \tag{13}$$

and

$$\theta(s)=\left[G_{\theta F}(s)G_{F\zeta}(s)+G_{\theta M}(s)G_{M\zeta}(s)\right]\zeta(s) \tag{14}$$

respectively.

Expressing these relations in terms of transfer functions, one obtains

$$\begin{aligned} G_{Z\zeta}(s)&=\frac{z(s)}{\zeta(s)} \\ &=G_{ZF}(s)G_{F\zeta}(s)+G_{ZM}(s)G_{M\zeta}(s) \end{aligned} \tag{15}$$

and

$$G_{\theta\zeta}(s) = \frac{\theta(s)}{\zeta(s)} \qquad (16)$$
$$= G_{\theta F}(s)G_{F\zeta}(s) + G_{\theta M}(s)G_{M\zeta}(s)$$

For a sinusoidal wave input, where $\zeta = \zeta_a \sin(\omega_e t)$, it can be shown that when steady state conditions are reached, the heaving and pitching frequency responses,

$$z_{ss}(t) = z_a \sin(\omega_e t + \phi_z)$$

$$\theta_{ss}(t) = \theta_a \sin(\omega_e t + \phi_\theta),$$

can be calculated by replacing the Laplace variable *s* in the transfer function Eqs. (15) and (16) by $j\omega_e$ such that

$$z_a = \left|G_{Z\zeta}(j\omega_e)\right|\zeta_a, \quad \phi_z = \angle G_{Z\zeta}(j\omega_e) \qquad (17)$$

$$\theta_a = \left|G_{\theta\zeta}(j\omega_e)\right|\zeta_a, \quad \phi_\theta = \angle G_{\theta\zeta}(j\omega_e) \qquad (18)$$

Here, $j = \sqrt{-1}$.

On this basis, $\left|G_{Z\zeta}(j\omega_e)\right|$ and $\left|G_{\theta\zeta}(j\omega_e)\right|$ are amplitude ratios of heave and pitch resulting from the encountered wave. Similarly, $\angle G_{Z\zeta}(j\omega_e)$ and $\angle G_{\theta\zeta}(j\omega_e)$ are the heave and pitch phase-shift with respect to the encountered wave. The amplitude ratios and phase shifts are functions of the encountered wave frequency and therefore, can be conveniently presented in graphical form as illustrated later in the paper. These figures show the heave amplitude ratio versus frequency of encounter. Given the wave amplitude and frequency for a specific seaway, the resulting heave amplitude can be read directly from such a graph.

3.1 Vehicle Parameters

The inertia, damping, and stiffness coefficients in Eq. (3) are defined in terms of s*trip theory* as presented by Krovin-Kroukovsky and Jacobs (1957). With this method, the vehicle is divided into a finite number of transverse sections along its length. The inertia, damping, and stiffness coefficients for each section are considered constant and are denoted by the subscript *n*. The overall vehicle coefficients are determined by integrating the section coefficients over the vehicle length, L. The definitions of Eq. (3) coefficients are summarized in Table 1.

Table 1 Hydrodynamic Coefficient Definitions

Term	Definition
$m = \int_L m_n d\xi$	Vehicle mass
$a_z = \int_L a_n d\xi$	Added mass (the mass of water excited by the vehicle's motion),

$b = \int_L b_n d\xi$	Heave damping coefficient
$c = \int_L c_n d\xi$	Heave stiffness coefficient
$d = -\int_L a_n \xi d\xi$	Pitch inertia coupling term
$e = -\int_L b_n \xi d\xi + ua_z$	Pitch damping coupling term
$h = -\int_L c_n \xi d\xi + ub$	Pitch stiffness coupling term
$I_{yy} = \int_L m_n \xi^2 d\xi$	Pitch axis moment of inertia
$A_{yy} = \int_L a_n \xi^2 d\xi$	Added-mass of inertia
$B = \int_L b_n \xi^2 d\xi$	Pitch damping coefficient
$C = \int_L c_n \xi^2 d\xi - uE$	Pitch stiffness coefficient
$D = d$	Heave inertia coupling term
$E = -\int_L b_n \xi d\xi - ua_z$	Heave damping coupling term
$H = -\int_L c_n \xi d\xi$	Heave Stiffness coupling term

The term m_n in Table 1 is sectional vehicle mass. The term a_n is the sectional added mass and can be determined from

$$a_n = \frac{1}{8} k_a \rho \pi B_n^2 \tag{19}$$

where ρ is the mass density, B_n is the vehicle's sectional beam width, and k_a is the added mass coefficient. k_a is a function draft/beam ratio, shape of the hull's cross sectional area, and the wave frequency, . Values of k_a for various hall cross section shapes can be found in Lewis (1929). The term b_n is the sectional damping coefficient and can be determined from

$$b_n = \frac{\rho g^2 R_z{}^2}{\omega_e^3} \tag{20}$$

where g is the gravitational constant and R_z is the ratio of radiated wave amplitude to heave motion amplitude. The expression for R_z used in this work was derived in Yamamoto et. al. (1980) and is given by

$$R_z = 2e^{-\frac{z_{ref}\omega_e^2}{g}} \sin\left(\frac{B_n \omega_e^2}{g}\right) \tag{21}$$

The term z_{ref} is a sectional reference draft of the wetted surface.

The sectional stiffness coefficient, c_n, is associated to the buoyancy restoring force due to heave disturbances and is

$$c_n = \rho g B_n \tag{22}$$

3.2 Wave Induced Force and Moment

In general, the force induced by harmonic waves can be written as

$$\begin{aligned} F(t) &= F_c \cos(\omega_e t) + F_s \sin(\omega_e t) \\ &= F_0 \sin(\omega_e t + \phi_F) \end{aligned} \tag{23}$$

where F_0 is the force amplitude, defined by

$$F_0 = \sqrt{F_c^2 + F_s^2} \tag{24}$$

and ϕ_F is the phase shift of the force with respect to the wave. That is,

$$\phi_F = \tan^{-1}\left(\frac{F_c}{F_s}\right) \tag{25}$$

The force amplitude terms F_c and F_s are derived in Goldstein (2001), Meirovitch (2010) and Benaroya (2004) and can be expressed as

$$F_c(\omega_e) = \zeta_a e^{-Kz} f_c(\omega_e) \tag{26}$$

$$f_c(\omega_e) = \int_L (c_n - \omega_e^2 a_n) \sin(K\xi) d\xi + \omega_e \int_L (b_n - u \frac{da_n}{d\xi}) \cos(K\xi) d\xi \tag{27}$$

$$F_s(\omega_e) = \zeta_a e^{-Kz} f_s(\omega_e) \tag{28}$$

$$f_s(\omega_e) = \int_L (c_n - \omega_e^2 a_n) \cos(K\xi) d\xi - \omega_e \int_L (b_n - u \frac{da_n}{d\xi}) \sin(K\xi) d\xi \tag{29}$$

Substituting Eqs. (26) and (28) into Eq. (24) and dividing the resulting equation by the wave amplitude, one obtains an expression for the amplitude ratios of force to wave. That is,

$$\frac{F_0}{\zeta_a}(\omega_e) = e^{-Kz}\sqrt{f_c^2 + f_s^2} \tag{30}$$

Similarly, substituting Eqs. (26) and (28) into Eq. (25) one obtains an expression for the phase shaft between the force and wave which is,

$$\phi_F(\omega_e) = \tan^{-1}\left(\frac{f_c}{f_s}\right) \tag{31}$$

The frequency transfer function between the wave induced force and the encountered waves can now be written as

$$G_{F\zeta}(j\omega_e) = \frac{F_0}{\zeta_a}e^{j\phi_F} \tag{32}$$

In a similar fashion, the moment induced by harmonic waves can be written as

$$\begin{aligned} M(t) &= M_c\cos(\omega_e t) + M_s\sin(\omega_e t) \\ &= M_0\sin(\omega_e t + \phi_M) \end{aligned} \tag{33}$$

where M_0 is the moment amplitude defined by

$$M_0 = \sqrt{M_c^2 + M_s^2} \tag{34}$$

and ϕ_M is the phase shift of the moment with respect to the wave, and is expressed as

$$\phi_M = \tan^{-1}\left(\frac{M_c}{M_s}\right) \tag{35}$$

The moment amplitude terms M_c and M_s can be expressed as

$$M_c(\omega_e) = \zeta_a e^{-Kz} m_c(\omega_e) \tag{36}$$

$$m_c(\omega_e) = \int_L (c_n - \omega_e^2 a_n)\xi\sin(K\xi)d\xi + \omega_e\int_L (b_n - u\frac{da_n}{d\xi})\xi\cos(K\xi)d\xi \tag{37}$$

$$M_s(\omega_e) = \zeta_a e^{-Kz} m_s(\omega_e) \tag{38}$$

$$m_s(\omega_e) = \int_L (c_n - \omega_e^2 a_n)\xi\cos(K\xi)d\xi - \omega_e\int_L (b_n - u\frac{da_n}{d\xi})\xi\sin(K\xi)d\xi \tag{39}$$

Substituting Eqs. (36) and (38) into Eq. (34) and dividing the resulting by the wave amplitude, an expression for the amplitude ratios of moment to wave is obtained. That expression is

$$\frac{M_0}{\zeta_a}(\omega_e) = e^{-Kz}\sqrt{m_c^2 + m_s^2} \tag{40}$$

Similarly, substituting Eqs. (36) and (38) into Eq. (35) an expression for the phase shaft between the moment and wave is found to be.

$$\phi_m(\omega_e) = \tan^{-1}\left(\frac{m_c}{m_s}\right) \tag{41}$$

The frequency transfer function between the wave induced moment and the encountered waves can now be written as follows:

$$G_{M\zeta}(j\omega_e) = \frac{M_0}{\zeta_a} e^{j\phi_M} \tag{42}$$

Eqs. (32) and (42) can now used with Eqs. (16) and (17) to develop the frequency response model.

4.0 Experiment

To validate the model developed above, the wave-structure interaction of a small, shallow-draft, flat-bottom boat was investigated in the 380 foot towing tank of the U. S. Naval Academy's Hydromechanics Laboratory, described in http://usna.edu/Hydromechanics/homepage.html. The test boat had the following characteristics:

Length = 3.05m

Beam = 1.142 m at L/2

Mass = 49.9 kg

To simulate the effects of onboard personnel and ancillary equipment, 120 kg of weights were added in the aft section of the boat as sketched in Figure 3. Additionally, the boat was attached at its center to the towing carriage via a set of heave posts which rode freely when any waves were encountered. The overall mass of the heave post was 17.4 kg, making the overall mass of the boat 187.3 kg. Figure 4 shows a schematic of the Naval Academy Towing Tank.

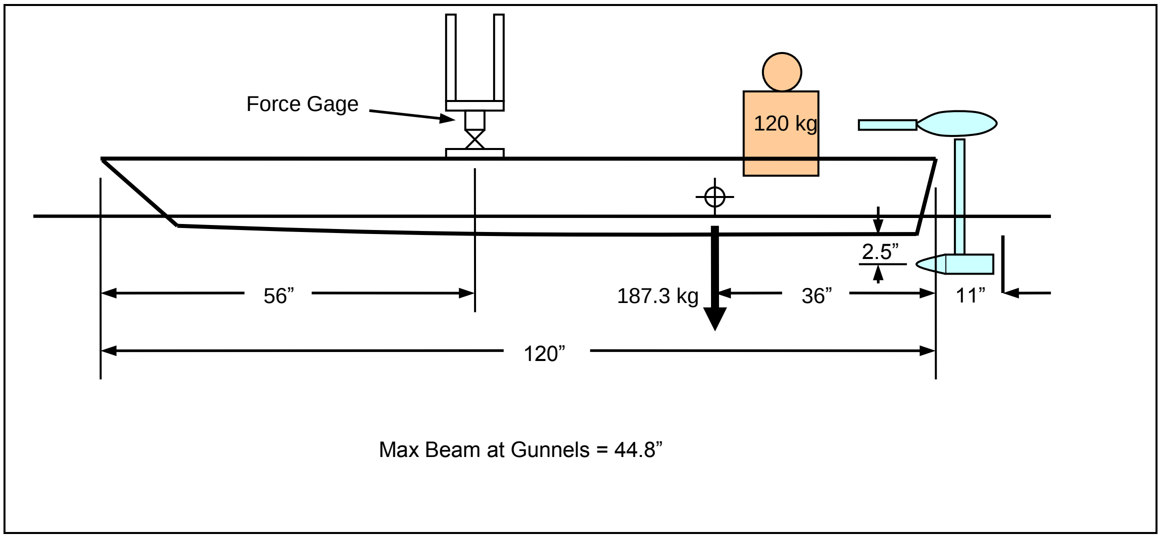

Fig. 3 Schematic of Test Boat

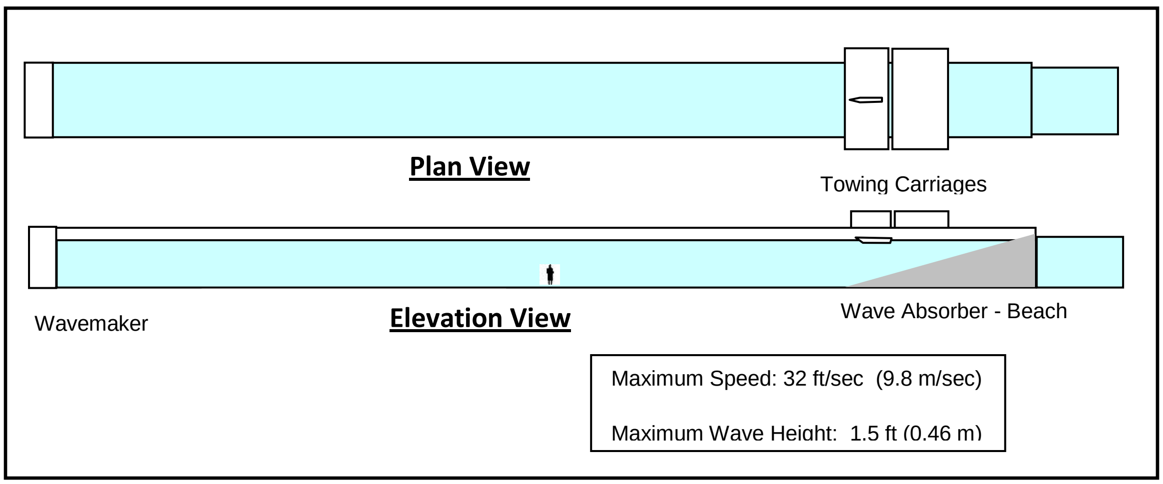

Fig. 4 Schematic Showing Plan and Elevation Views of Naval Academy Tow Tank

The 380-foot towing tank is large enough to test ship models up to 25 feet in length and weighing several thousand pounds. The tank length allows testing at high speeds. The tank is outfitted with two towing carriages, a dual-flap wavemaker and specialized equipment for measuring resistance, seakeeping and maneuvering characteristics of all types of marine vehicles and ocean platforms. The test boat was attached to the towing carriage as seen below in Figures 5 and 6.

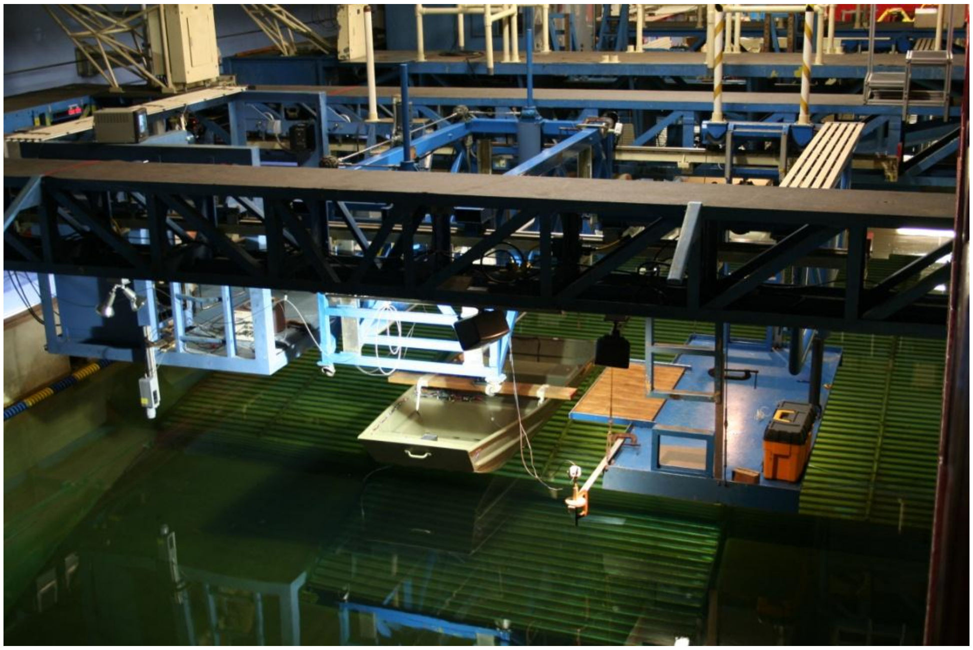

Fig. 5 Photograph of the Test Boat Connected to Towing Carriage in Towing Tank

Fig. 6 View of the Test Boat from the Carriage

Twenty experimental runs were made, while varying three parameters: wave height, wave frequency, and ship speed. A summary of the experimental runs is given in Table 2.

Table 2 Experimental Heave-to-Wave Amplitude Ratios

Boat Velocity (Kts)	Wave Frequency (Hz)	Encountered Wave Frequency (Hz)	Measured Heave-to-Wave Amplitude Ratio, $\left(\frac{z_a}{\zeta_a}\right)$		
			1" Wave Height	2" Wave Height	3" Wave Height
3	0.4	0.56	0.992	1.088	1.105
3	0.7	1.18	0.752	0.712	-
3	1.0	1.99	0.140	0.109	-
4	0.4	0.61	1.035	1.085	1.141
4	0.7	1.35	0.510	0.723	-
4	1.0	2.32	0.152	0.180	-
6	0.4	0.72	1.196	1.361	1.397
6	0.7	1.67	0.885	0.860	-
6	1.0	2.98	-	0.275	-

5.0 Results

In this section, experimental results are used to validate the dynamic model developed herein. The boat heave and wave height for each experimental run was measured and recorded in time. To illustrate the experimental results, the heave time responses at boat speeds of 3, 4, and 6 Kts to 1 inch, 0.4 Hz harmonic waves are shown in Figures 7 through 9.

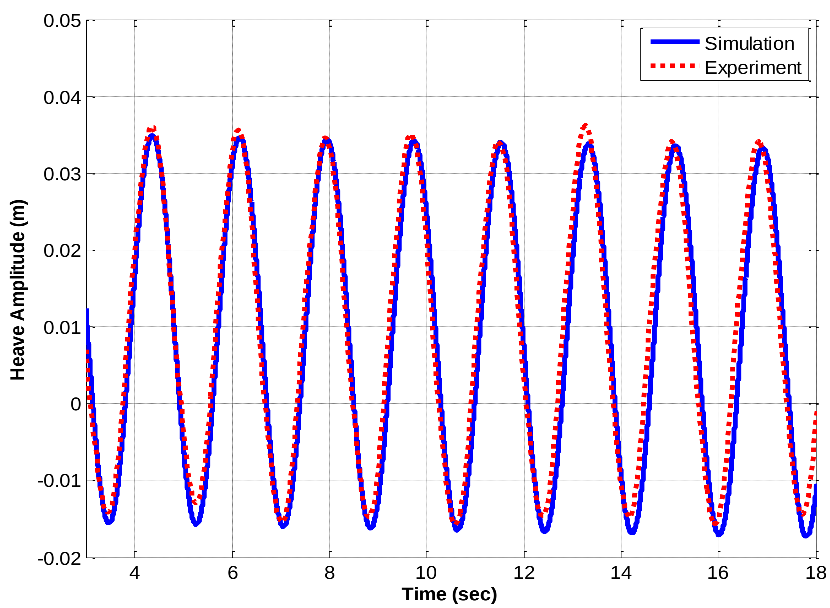

Fig. 7 Heave Time Response to 1 Inch - 0.4 Hz Waves, Vehicle Velocity = 3 Kts

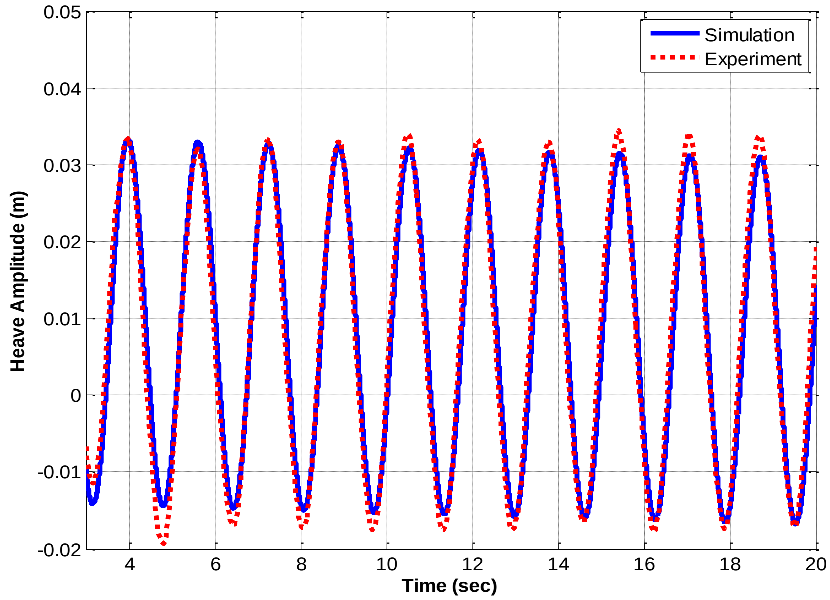

Fig. 8 Heave Time Response to 1 Inch - 0.4 Hz Waves, Vehicle Velocity = 4 Kts

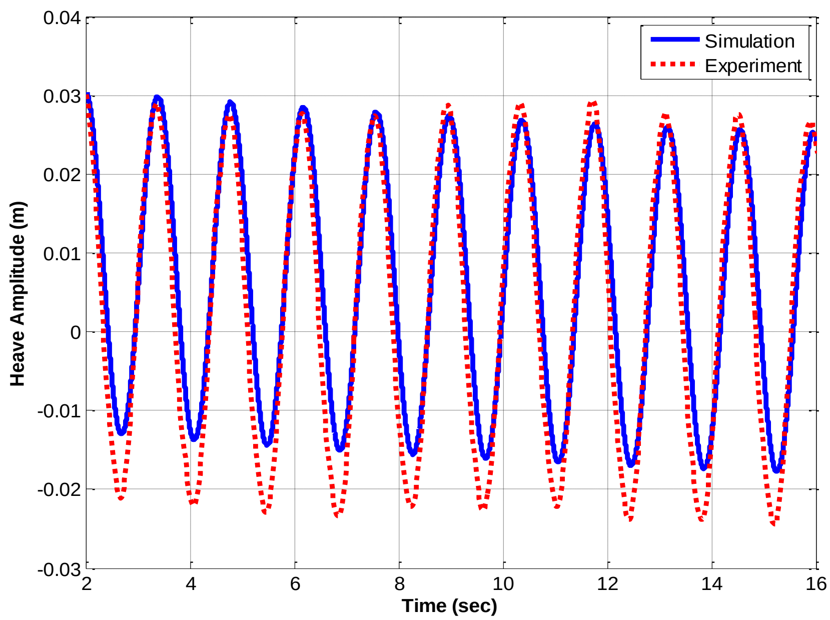

Fig. 9 Heave Time Response to 1 Inch - 0.4 Hz Waves, Vehicle Velocity = 6 Kts

The experimental results show that the heave response to regular harmonic waves is also harmonic with frequency equal to the encountered wave as expected. Also shown in Figures 7 through 9 are results of heave time response simulations obtained from the heave dynamic model in Eq. (13). The measured time history of the wave height was used as input for the simulations. As can be seen, the time simulation and the experimental results compare well.

Knowing that the boat's response to regular harmonic waves is also harmonic, it is highly advantageous to represent the boat's behavior as a frequency response. The frequency response conveniently characterizes the boat's response to a wider range of seaway conditions, as opposed to time responses. From the experimental data, the ratio of the boat heave amplitude, z_a, and the wave amplitude, ζ_a, were calculated and shown in Table 2.

The experimental heave-to-wave amplitude ratios are plotted as a function of the frequency of encounter (ω_e) for the three boat velocities in Figures 10 to 12. The figures show that the heave amplitude ratio is fairly independent to wave height changes at a given frequency. This result supports the linearity assumption of the dynamics described in Eq. (3) at a specified input frequency.

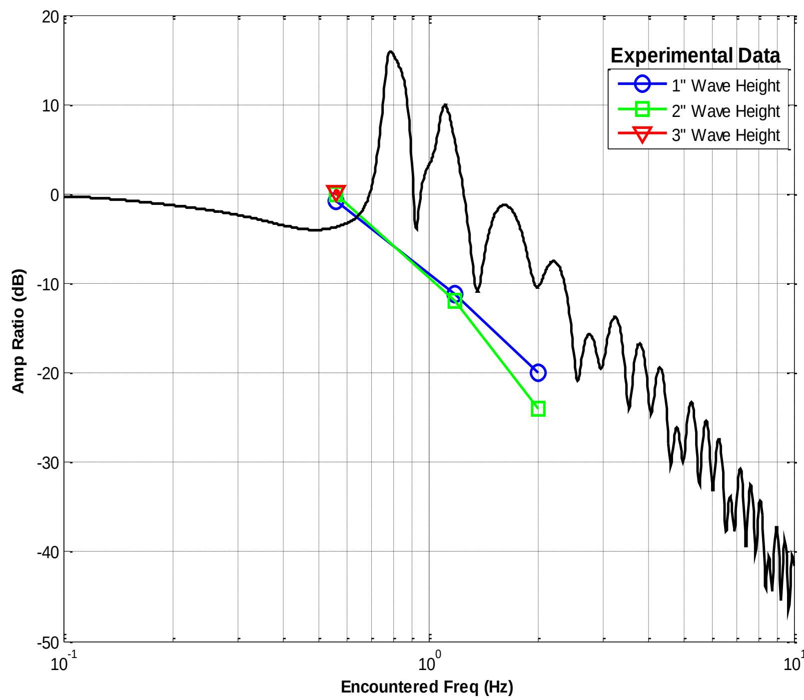

Fig. 10 Heave Frequency Response to Wave Input, Vehicle Velocity = 3 Kts

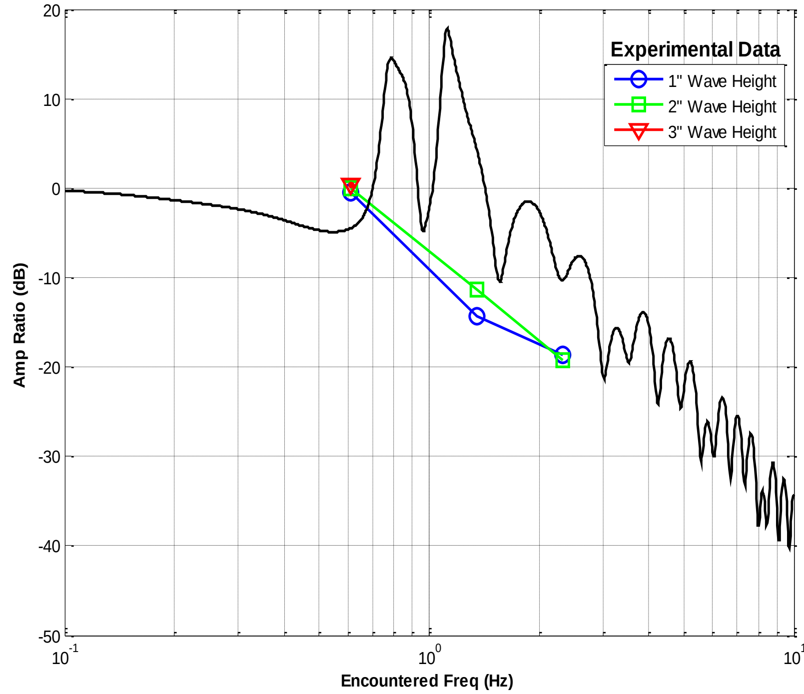

Fig. 11 Heave Frequency Response to Wave Input, Vehicle Velocity = 4 Kts

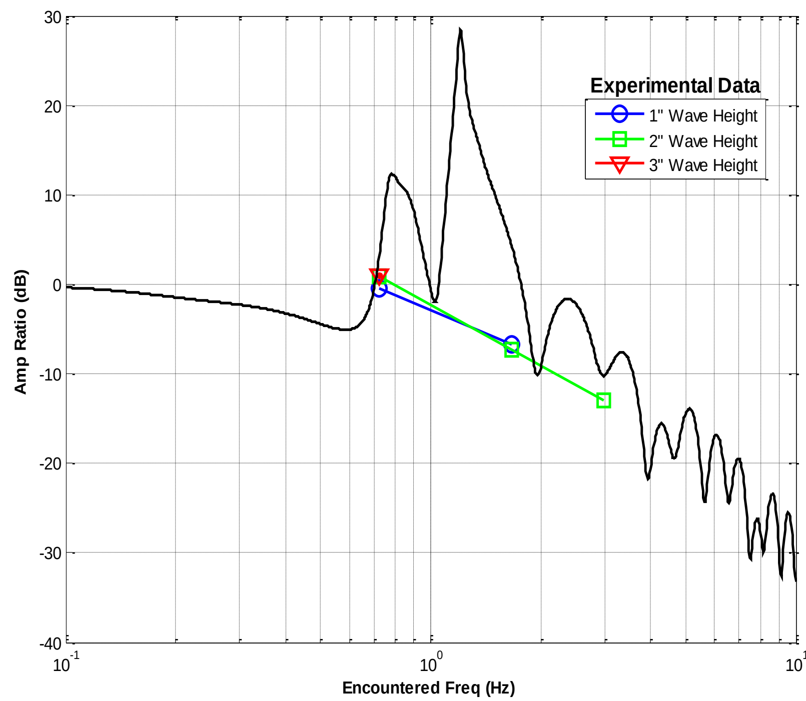

Fig. 12 Heave Frequency Response to Wave Input, Vehicle Velocity = 6 Kts

The frequency response amplitude ratio $\left|G_{Z\zeta}(j\omega_e)\right|$ in Eq. (17) was calculated for the test boat, and is shown superimposed on the experimental data in Figures 10 through 12. The frequency domain model shows the characteristics associated with a fourth-order linear physical system. That is, constant low frequency amplitude with resonate peaks around the natural frequency, and a -40 dB per decade slope at high frequencies. The heave natural frequency can be calculated by

$$\omega_n = 2\pi f_n = \sqrt{\frac{c}{m + a_z}} \tag{43}$$

For the test boat, the natural frequency is approximately equal to f_n = 0.64 Hz. The peaks and valleys observed at frequencies greater than ω_n correspond to nonlinearities in the buoyancy force and moment. Buoyancy depends heavily on the relationship between wave length and boat length. Buoyancy will decrease when the boat length is an integer multiple of the wave length and will increase when it is not. Therefore, as the wave frequency increases and the wave length decreases, the buoyancy force and moment will rise and fall.

In general, the frequency domain model corresponds well with the experimental data, especially at low frequencies. Any discrepancies observed between the model and data can be attributed to inaccuracies in the model's added mass and damping coefficient terms. Expressions for these terms were derived from interpolations of experimental data and therefore, inherently have a degree of uncertainty. In addition, experimental error associated with the towing tank experiments may have also contributed to any discrepancies.

6.0 Conclusions

In this paper, a model for the coupled heave and pitch dynamics of surface vehicles is developed using a novel approach of representing the ship dynamics in the frequency domain. The model is based on theoretical first principles involving highly nonlinear equations that depend on the wave frequency. Compared to time domain modeling and simulation approaches, the frequency domain model provides greater insight to the surface vehicle's behavior over a wider operating range. The theoretical model was validated by experimental data that was obtained from towing tank experiment. The test boat has a 3-meter shallow draft and a flat bottom. The theoretical model results compare well with the experimental data.

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